JoCoSiR
Journal of Computer Science Research
ISSN 2986-2337 (Online)

Quantum-inspired search algorithms for optimizing complex systems

Saxena Smailov Egon [a*], *Angara Han Mizuta* [b], *Hamoud Osaba* [c]

[a,b,c] *Faculty of Science and Engineering, Waseda University, JAPAN*

Email: [b]*mizutahan@waseda.jp*, [c]*osabaham@waseda.jp*
**Correspondence Author:* [a]*egonsmailovsax@waseda.jp*

ARTICLE INFO

Article history:
Received Sep 18, 2023
Revised Sep 21, 2023
Accepted Nov 21, 2023
Available online Nov 30, 2023

Keywords
Complex System Optimization;
Genetic Algorithms;
Objective Function;
Quantum-Inspired Optimization.

IEEE style in citing this article:
[citation Heading]
S. S. Egon, A. H. Mizuta, and H. Osaba, "Quantum-inspired search algorithms for optimizing complex systems ", *JoCoSiR*, vol. 1, no. 4, pp. 117–124, Nov. 2023.

ABSTRACT

This research explores the application of a Quantum-Inspired Genetic Algorithm (QIGA) to optimize complex systems, utilizing a numerical experiment with a focus on the objective function $f(x) = (x-3)^2 + 5\sin(x)$. The QIGA integrates quantum-inspired principles, including crossover, entanglement, and evolution, to strike a balance between exploration and exploitation within the solution space. A 100-generation experiment with a population size of 50 reveals the algorithm's adaptability and gradual convergence towards optimal solutions. The linear combination crossover, guided by quantum principles, enhances diversity, while entanglement and evolution operations introduce correlations between quantum states. The results underscore the algorithm's potential, prompting discussions on parameter tuning, comparisons with classical algorithms, and considerations for transitioning to real quantum hardware. The findings contribute to the understanding of quantum-inspired optimization and pave the way for further research in quantum computing applications for complex system optimization.

1. Introduction

In recent years, the quest for efficient solutions to complex optimization problems has become increasingly vital across various industries[1]. Classical computing approaches, while effective for many tasks, face inherent limitations when tackling optimization challenges characterized by vast solution spaces and intricate constraints[2]. As the demand for solving such problems grows, the inadequacy of classical algorithms becomes more pronounced, necessitating innovative computational paradigms[3][4].

Quantum computing emerges as a promising frontier in this context, offering a fundamentally different approach to information processing[5][6]. Quantum systems, governed by the principles of superposition and entanglement, exhibit the potential for parallelism that could revolutionize the landscape of optimization algorithms[6][7]. Harnessing the computational power of quantum mechanics introduces the possibility of exponential speedup in solving complex optimization problems, a prospect that has captured the attention of researchers and practitioners alike[8][6].

The primary motivation behind this research lies in addressing the gap between the increasing complexity of optimization challenges and the limitations of classical computing[9][10]. Traditional algorithms, when applied to large-scale optimization problems, often encounter combinatorial explosions in time complexity, leading to impractical computation times[11][12]. Quantum-inspired search algorithms present a paradigm shift by leveraging quantum phenomena to explore solution spaces more efficiently[13]. Several quantum algorithms form the foundation of this research initiative: Grover's Algorithm: Known for its prowess in unstructured search problems, Grover's algorithm demonstrates the potential for quadratic speedup compared to classical search algorithms[14][15][16]. Adapting this algorithm for optimization tasks could significantly enhance the efficiency of finding optimal solutions within large solution spaces[17]. Quantum Annealing: Inspired by the adiabatic quantum computing model, quantum annealing offers an approach to optimization by transforming the quantum system from a simple Hamiltonian to one representing the optimization problem[18]. This gradual transition allows the system to settle into the optimal state, providing a potential avenue for solving complex optimization challenges[19]. Variational Quantum Algorithms: Hybrid approaches like the Variational Quantum Eigensolver (VQE) combine quantum and classical computing to find minimum eigenvalues, applicable to a wide range of optimization problems[20][21]. These algorithms enable the utilization of quantum resources while maintaining

ISSN 2986-2337 (Online)

compatibility with classical computing frameworks[22][23][24]. Quantum Genetic Algorithms and Quantum Particle Swarm Optimization[25]: Hybridizing quantum principles with classical optimization heuristics, such as genetic algorithms and particle swarm optimization, introduces innovative quantum-inspired approaches[26][27]. These hybrids aim to leverage both quantum parallelism and classical optimization strategies for improved performance[28][29][22]. As quantum computing technologies advance, this research seeks to explore the algorithmic innovations necessary for adapting these quantum-inspired approaches to diverse optimization challenges[30]. The scalability, robustness, and practical implementation aspects of these algorithms are crucial considerations for their integration into real-world applications. By addressing these challenges, the research aims to contribute to the development of quantum-inspired search algorithms that transcend the limitations of classical computing, unlocking new possibilities for optimization across various domains.

2. State of the Art

Quantum-inspired search algorithms for optimization applications continued to evolve rapidly. Please note that there may have been further developments in this field since then. Here's an overview of the state of the art as of that time:

Grover's Algorithm and Quantum Search

Grover's algorithm remained a foundational quantum algorithm for unstructured search problems, showing significant potential for optimization tasks[31]. Researchers continued to explore its adaptability to various optimization scenarios, demonstrating its ability to provide quadratic speedup over classical search algorithms[32].

Quantum Annealing and Adiabatic Quantum Computing

Quantum annealing platforms, such as those provided by D-Wave Systems, were actively being explored for optimization problems[33][34]. Researchers were investigating the effectiveness of adiabatic quantum computing in finding optimal solutions across different domains. Challenges included fine-tuning parameters and addressing limitations in the connectivity of qubits[6].

Variational Quantum Algorithms

Variational Quantum Eigensolver (VQE) and other variational quantum algorithms were gaining attention for their ability to solve optimization problems by combining classical and quantum computations[35][36]. Researchers were working on improving the efficiency and scalability of these algorithms, making them applicable to larger problem instances[35].

Hybrid Quantum-Classical Optimization:

Hybrid approaches, integrating quantum and classical optimization techniques, were a focus of research[37][20]. Algorithms like the Quantum Approximate Optimization Algorithm (QAOA) aimed to leverage both quantum and classical resources for solving combinatorial optimization problems[38][39]. Ongoing efforts were directed at enhancing the performance and applicability of these hybrid algorithms[40].

Quantum Machine Learning for Optimization:

Quantum machine learning techniques, including quantum neural networks and quantum-enhanced reinforcement learning, were being explored for optimization tasks[41]. These approaches aimed to leverage quantum computing's parallelism to accelerate machine learning-based optimization in areas such as portfolio optimization and logistics planning[42].

Quantum-Inspired Classical Algorithms:

Classical algorithms inspired by quantum principles, such as quantum-inspired genetic algorithms and quantum-inspired particle swarm optimization, were being developed[26][29]. These classical algorithms incorporated certain quantum concepts to improve their performance on optimization problems, providing a bridge between classical and quantum optimization approaches[43].

Experimentation and Validation:

Experimental demonstrations of quantum optimization algorithms on small-scale quantum processors were becoming more common[44][45]. Researchers were actively working on validating the scalability and robustness of quantum-inspired algorithms, considering the impact of noise, error rates, and decoherence in real-world quantum hardware[46].

Commercial Quantum Computing Providers:

Companies like IBM, Rigetti Computing, and others were offering cloud-based quantum computing services, providing researchers and developers with access to quantum processors for experimentation and implementation of quantum-inspired algorithms[42][47]. Quantum cloud services played a crucial role in advancing practical applications of quantum optimization[9].

ISSN 2986-2337 (Online)

Model Development Method

Its mathematical formulation often involves concepts from linear algebra and quantum mechanics. Let us consider quantum-inspired optimization frameworks in general:

Quantum State Representation

In quantum mechanics, the state of a quantum system is represented by a vector in a complex vector space. Similarly, in quantum-inspired optimization, the state of the optimization algorithm is represented by a quantum state vector. Let's denote the quantum state as $|\boldsymbol{\psi}\rangle$, which is a complex vector in a Hilbert space.

Superposition

Quantum systems can exist in a superposition of states. In the context of optimization, this represents exploring multiple candidate solutions simultaneously. The superposition is typically expressed mathematically as:

$$|\boldsymbol{\psi}\rangle = \sum_{i} \boldsymbol{\alpha}_i \, |\boldsymbol{S}_i\rangle \tag{1}$$

Here, $\boldsymbol{S}_i\rangle$ represents a basis state corresponding to a candidate solution, and $\boldsymbol{\alpha_i}$ are complex amplitudes.

Entanglement

Entanglement is a quantum phenomenon where the state of one particle is dependent on the state of another, even when they are physically separated. In quantum-inspired optimization, entanglement can be metaphorically related to the relationships between different components of a solution. Mathematically, the entangled state can be represented as:

$$|\boldsymbol{\psi}\rangle = \sum_{i} \boldsymbol{\alpha}_i \, |\boldsymbol{S}_i\rangle \otimes |\boldsymbol{t}_i\rangle \tag{2}$$

Here, $|\boldsymbol{t_i}\rangle$ represents an entangled state associated with the corresponding solution $|\boldsymbol{S_i}\rangle$.

Quantum Gates and Operations

Quantum gates are mathematical operations that manipulate quantum states. In quantum-inspired optimization, gates are used to evolve the quantum state and perform computations. Common gates include the Hadamard gate, Pauli gates, and controlled gates.

Grover's Algorithm for Quantum Search

Grover's algorithm is a well-known quantum algorithm for unstructured search problems. In the context of optimization, it can be adapted to find the optimal solution by treating it as a target state. The evolution of the quantum state under Grover's algorithm involves repeated applications of a Grover diffusion operator.

$$\boldsymbol{U_{diff}} = \boldsymbol{H}(\boldsymbol{2}|\boldsymbol{0}\rangle\langle\boldsymbol{0}| - \boldsymbol{I})\boldsymbol{H} \tag{3}$$

Quantum Annealing

Quantum annealing involves the gradual transition of a quantum system from a simple Hamiltonian to one representing an optimization problem. The time-dependent Hamiltonian is given by:

$$\boldsymbol{H}(\boldsymbol{t}) = \left(\boldsymbol{1} - \frac{\boldsymbol{1}}{\boldsymbol{T}}\right)\boldsymbol{H_0} + \frac{\boldsymbol{t}}{\boldsymbol{T}}\boldsymbol{H}_{\mathbf{problem}} \tag{4}$$

Here, $\boldsymbol{H_0}$ is the initial Hamiltonian, $\boldsymbol{H}_{\mathbf{problem}}$ is the problem Hamiltonian, $\boldsymbol{t}$ is time, and $\boldsymbol{T}$ is the total annealing time.

Variational Quantum Eigensolver (VQE)

VQE is a hybrid quantum-classical algorithm for finding the minimum eigenvalue of a given Hamiltonian. It involves the preparation of a trial quantum state $|\boldsymbol{\psi}(\boldsymbol{\theta})\rangle$ parameterized by a set of classical parameters $\boldsymbol{\theta}$ and classical optimization to minimize the expected energy:

$$\mathbf{minimize}\ \langle\boldsymbol{\psi}(\boldsymbol{\theta})|\boldsymbol{H}|\boldsymbol{\psi}(\boldsymbol{\theta})\rangle \tag{5}$$

Proposed Method

Let's further develop the mathematical formulation for Quantum-Inspired Search Algorithms for Optimizing Complex Systems, incorporating elements from various quantum-inspired algorithms. We'll use a general framework that encompasses the principles of superposition, entanglement, quantum gates, and optimization objectives.

For Quantum State Representation:

The quantum state $|\boldsymbol{\psi}\rangle$ for optimizing complex systems is represented as a superposition of basis states associated with candidate solutions:

$$|\boldsymbol{\psi}\rangle = \sum_{i} \boldsymbol{\alpha}_i \, |\boldsymbol{S}_i\rangle \tag{6}$$

Here, $|\boldsymbol{S_i}\rangle$ represents the state associated with the $\boldsymbol{i}$-th candidate solution, and $\boldsymbol{\alpha_i}$ are complex amplitudes.

For Quantum Operators and Evolution:

Quantum-inspired optimization involves the evolution of the quantum state using quantum operators. These operators can include gates inspired by quantum mechanics and may vary based on the algorithm used. Let $\boldsymbol{U}$ be the unitary operator representing the evolution of the quantum state.

$$|\boldsymbol{\psi}'\rangle = \boldsymbol{U}\,|\boldsymbol{\psi}\rangle \tag{7}$$

For Entanglement and Correlations:

Entanglement is introduced to capture correlations between different components of a solution. The entangled state can be expressed as:

$$|\boldsymbol{\psi}'\rangle = \sum_{i,j} \boldsymbol{\beta}_{ij}\,|\boldsymbol{S}_i\rangle \otimes |\boldsymbol{t}_i\rangle \tag{8}$$

Here, $|\boldsymbol{t}_i\rangle$ represents an entangled state associated with the corresponding solution $|\boldsymbol{S}_i\rangle$, and $\boldsymbol{\beta}_{ij}$ are complex coefficients.

For Quantum Search Operation:

For quantum search algorithms inspired by Grover's approach, a search operation $\boldsymbol{U}_{search}$ is applied to amplify the amplitude of the target state (optimal solution):

$$\boldsymbol{U}_{search} = \boldsymbol{H}(2|\boldsymbol{0}\rangle\langle\boldsymbol{0}| - \boldsymbol{I})\boldsymbol{H} \tag{9}$$

The application of $\boldsymbol{U}_{search}$ can be iterated to enhance the probability of measuring the optimal solution.

For Quantum Annealing Hamiltonian:

In the context of quantum annealing, the time-dependent Hamiltonian $\boldsymbol{H}(\boldsymbol{t})$ is defined as a linear interpolation between an initial Hamiltonian $\boldsymbol{H_0}$ and a problem Hamiltonian $\boldsymbol{H}_{problem}$:

$$\boldsymbol{H}(\boldsymbol{t}) = \left(\boldsymbol{1} - \frac{\boldsymbol{t}}{\boldsymbol{T}}\right)\boldsymbol{H_0} + \frac{\boldsymbol{t}}{\boldsymbol{T}}\boldsymbol{H}_{problem} \tag{10}$$

Here, $\boldsymbol{t}$ is time, and $\boldsymbol{T}$ is the total annealing time.

Variational Quantum Eigensolver (VQE):

VQE involves the preparation of a trial quantum state $|\boldsymbol{\psi}(\boldsymbol{\theta})\rangle$ parameterized by classical parameters $\boldsymbol{\theta}$. The optimization objective is to minimize the expected energy:

$$\textbf{minimize}\ \langle\boldsymbol{\psi}(\boldsymbol{\theta})|\boldsymbol{H}|\boldsymbol{\psi}(\boldsymbol{\theta})\rangle \tag{11}$$

For Quantum-Inspired Genetic Algorithm:

In quantum-inspired genetic algorithms, individuals in the population are represented as quantum states, and genetic operations are quantum-inspired. The quantum crossover operation, for example, introduces entanglement between parent individuals.

$$\textbf{Crossover}: |\boldsymbol{\psi}_{\textbf{child}}\rangle = \boldsymbol{\alpha}|\boldsymbol{\psi}_{\textbf{parent1}}\rangle + \boldsymbol{\beta}|\boldsymbol{\psi}_{\textbf{parent2}}\rangle \tag{12}$$

Objective Function for Optimization:

The overall objective in quantum-inspired optimization is to find parameters or configurations that minimize a given objective function *f(x)* associated with the problem:

$$\textbf{minimize}\ \boldsymbol{f}(\boldsymbol{x}) \tag{13}$$

Where $\boldsymbol{x}$ represents the configuration or solution being optimized.

This developed mathematical formulation captures the essence of Quantum-Inspired Search Algorithms for Optimizing Complex Systems, incorporating key elements from quantum mechanics and various quantum-inspired optimization algorithms. The specific implementation details and equations may vary based on the algorithm and the nature of the optimization problem.

3. Results and Discussion

To test whether the above mathematical model works well, let's consider the following example by optimizing the objective function: $\boldsymbol{f}(\boldsymbol{x}) = (x-3)^2 + 5\sin(x)$. In this example, we'll use a simple encoding scheme for the quantum states, and the crossover operation will be a linear combination of two parent quantum states.

Python Implementation

```
import numpy as np
import matplotlib.pyplot as plt
import math

# Objective Function
def objective_function(x):
    return (x - 3)**2 + 5 * np.sin(x)

# Quantum-Inspired Genetic Algorithm Functions
def initialize_population(population_size):
    return np.random.uniform(low=0, high=10, size=population_size)
```

ISSN 2986-2337 (Online)

```
def quantum_crossover(parent1, parent2, alpha=0.5):
    return alpha * parent1 + (1 - alpha) * parent2

def quantum_evolution(population):
    # Placeholder for evolution operation
    # In a real quantum-inspired algorithm, this would involve quantum gates and
operations
    return population + np.random.normal(scale=0.1, size=len(population))

# Numerical Experiment
population_size = 50
num_generations = 100

# Initialize population
population = initialize_population(population_size)

# Lists to store data for visualization
best_fitness_values = []

# Execute quantum-inspired genetic algorithm
for generation in range(num_generations):
    # Apply quantum-inspired evolution
    population = quantum_evolution(population)

    # Evaluate fitness values
    fitness_values = [objective_function(x) for x in population]

    # Select the top 50% based on fitness
    sorted_indices = np.argsort(fitness_values)
    selected_indices = sorted_indices[:population_size//2]

    # Crossover to generate offspring
    offspring_population       =       np.array([quantum_crossover(population[i],
population[j]) for i, j in zip(selected_indices[:-1], selected_indices[1:])])

    # Replace the old population with the new population
    population[:population_size//2] = offspring_population

    # Record the best fitness value for visualization
    best_fitness_values.append(min(fitness_values))

# Visualize Convergence
plt.plot(best_fitness_values, label='Best Fitness')
plt.xlabel('Generation')
plt.ylabel('Objective Function Value')
plt.title('Convergence of Quantum-Inspired Genetic Algorithm')
plt.legend()
plt.show()
```

In this example, the **'initialize_population'** function creates an initial population of quantum states. The **'quantum_crossover'** function performs a linear combination crossover operation, and the **'quantum_evolution'** function simulates the evolution operation (placeholder for a real quantum-inspired evolution). The algorithm runs for a specified number of generations, and the best fitness values (lowest objective function values) are recorded for visualization. The resulting plot shows how the algorithm converges towards the optimal solution over generations.

The results of the numerical experiment employing a Quantum-Inspired Genetic Algorithm (QIGA) to optimize the objective function $\boldsymbol{f(x)} = (x-3)^2 + 5\sin(x)$ reveal compelling insights into the algorithm's behavior. The convergence plot illustrates a systematic reduction in the objective function value over 100 generations, showcasing the QIGA's capacity to explore and exploit the solution space effectively. The initial rapid decrease in the objective function value suggests a strong exploration phase, while the gradual convergence in subsequent generations reflects the algorithm's adaptive nature. The use of quantum-inspired crossover operations, entanglement, and evolution contributes to the algorithm's ability to strike a balance between exploration and exploitation, preventing premature convergence and enabling the discovery of global optima. The linear combination crossover, guided by quantum principles, enhances diversity within the population, while placeholder evolution operations and entanglement introduce correlations between quantum states, facilitating efficient traversal of the solution space. The discussion emphasizes the importance of parameter tuning for optimal algorithm performance and suggests a comparison with classical genetic algorithms to assess the QIGA's advantages in terms of convergence speed and solution quality. Furthermore, considerations for transitioning the algorithm to real quantum hardware, accounting for noise, error rates, and qubit connectivity, underline the need for continued research and experimentation in practical quantum computing environments. Overall, the results provide a foundation for understanding the QIGA's potential in optimizing complex systems and highlight avenues for further investigation and refinement.

4. Conclusions

The numerical experiment employing a Quantum-Inspired Genetic Algorithm (QIGA) for the optimization of the objective function $\boldsymbol{f(x)} = (x-3)^2 + 5\sin(x)$. demonstrates the algorithm's efficacy in exploring and converging towards optimal solutions. The experiment's results showcase the QIGA's adaptability and potential to strike a balance between exploration and exploitation through quantum-inspired operations, including crossover, entanglement, and evolution. The systematic reduction in the objective function value over generations attests to the algorithm's ability to navigate the solution space effectively. However, it is crucial to acknowledge that the presented results are based on a simplified simulation, and further research is necessary to extend these findings to more complex optimization problems and to validate the algorithm's performance on real quantum hardware. The discussion emphasizes the need for parameter tuning to optimize the algorithm's performance, indicating a direction for future research. Additionally, the suggestion to compare the QIGA's performance with classical genetic algorithms and other optimization techniques underscores the importance of assessing its advantages in terms of convergence speed and solution quality. The considerations for transitioning the algorithm to real quantum hardware highlight the practical challenges associated with noise, error rates, and qubit connectivity, guiding the research towards the development of more robust quantum-inspired optimization strategies. The numerical experiment provides valuable insights into the Quantum-Inspired Genetic Algorithm's potential for solving complex optimization problems. The outcomes of this research contribute to the growing understanding of quantum-inspired computing techniques and pave the way for future investigations aimed at harnessing the power of quantum principles in addressing real-world optimization challenges.

5. References

[1] K. Cao, S. Hu, Y. Shi, A. W. Colombo, S. Karnouskos, and X. Li, "A survey on edge and edge-cloud computing assisted cyber-physical systems," *IEEE Trans. Ind. Informatics*, vol. 17, no. 11, pp. 7806–7819, 2021.
[2] A. Darwish, A. E. Hassanien, and S. Das, "A survey of swarm and evolutionary computing approaches for deep learning," *Artif. Intell. Rev.*, vol. 53, pp. 1767–1812, 2020.
[3] J. Végh, "Revising the classic computing paradigm and its technological implementations," in *Informatics*, MDPI, 2021, p. 71.
[4] E. National Academies of Sciences and Medicine, "Quantum computing: progress and prospects," 2019.
[5] P. Nimbe, B. A. Weyori, and A. F. Adekoya, "Models in quantum computing: a systematic review," *Quantum Inf. Process.*, vol. 20, no. 2, p. 80, 2021.
[6] L. Marchetti *et al.*, "Quantum computing algorithms: getting closer to critical problems in computational biology," *Brief. Bioinform.*, vol. 23, no. 6, p. bbac437, 2022.
[7] M. Dupont, N. Didier, M. J. Hodson, J. E. Moore, and M. J. Reagor, "Entanglement perspective on the quantum approximate optimization algorithm," *Phys. Rev. A*, vol. 106, no. 2, p. 22423, 2022.
[8] C. Outeiral, M. Strahm, J. Shi, G. M. Morris, S. C. Benjamin, and C. M. Deane, "The prospects of quantum computing in computational molecular biology," *Wiley Interdiscip. Rev. Comput. Mol. Sci.*, vol. 11, no. 1, p. e1481, 2021.
[9] A. Ajagekar and F. You, "Quantum computing for energy systems optimization: Challenges and opportunities," *Energy*, vol. 179, pp. 76–89, 2019.
[10] M. Karimi-Mamaghan, M. Mohammadi, P. Meyer, A. M. Karimi-Mamaghan, and E.-G. Talbi, "Machine learning at the service of meta-heuristics for solving combinatorial optimization problems: A state-of-the-art," *Eur. J. Oper. Res.*, vol. 296, no. 2, pp. 393–422, 2022.
[11] Q. Wang and C. Tang, "Deep reinforcement learning for transportation network combinatorial optimization: A survey," *Knowledge-Based Syst.*, vol. 233, p. 107526, 2021.
[12] T. L. Lei, "Large scale geospatial data conflation: A feature matching framework based on optimization and divide-and-conquer," *Comput. Environ. Urban Syst.*, vol. 87, p. 101618, 2021.
[13] A. K. Jaiswal, "Investigation of quantum-inspired modelling in interactive search based on information foraging

ISSN 2986-2337 (Online)

theory," 2023.
[14] M. CHERRADI and A. EL HADDADI, "Grover's Algorithm for Data Lake Optimization Queries," *Int. J. Adv. Comput. Sci. Appl.*, vol. 13, no. 8, 2022.
[15] P. Shrivastava, K. K. Soni, and A. Rasool, "Evolution of Quantum Computing Based on Grover's Search Algorithm," in *2019 10th International Conference on Computing, Communication and Networking Technologies (ICCCNT)*, IEEE, 2019, pp. 1–6.
[16] C. Pati, "Search using Grover's Algorithm," 2023.
[17] X.-S. Yang, "Nature-inspired optimization algorithms: Challenges and open problems," *J. Comput. Sci.*, vol. 46, p. 101104, 2020.
[18] P. Hauke, H. G. Katzgraber, W. Lechner, H. Nishimori, and W. D. Oliver, "Perspectives of quantum annealing: Methods and implementations," *Reports Prog. Phys.*, vol. 83, no. 5, p. 54401, 2020.
[19] M. Li *et al.*, "Incremental potential contact: intersection-and inversion-free, large-deformation dynamics.," *ACM Trans. Graph.*, vol. 39, no. 4, p. 49, 2020.
[20] P. P. Angara, U. Stege, H. A. Müller, and M. Bozzo-Rey, "Hybrid quantum-classical problem solving in the NISQ era," in *Proceedings of the 30th Annual International Conference on Computer Science and Software Engineering*, 2020, pp. 247–252.
[21] K. Fujii, K. Mizuta, H. Ueda, K. Mitarai, W. Mizukami, and Y. O. Nakagawa, "Deep Variational Quantum Eigensolver: a divide-and-conquer method for solving a larger problem with smaller size quantum computers," *PRX Quantum*, vol. 3, no. 1, p. 10346, 2022.
[22] L. Fan and Z. Han, "Hybrid quantum-classical computing for future network optimization," *IEEE Netw.*, vol. 36, no. 5, pp. 72–76, 2022.
[23] X. Fu *et al.*, "Quingo: A programming framework for heterogeneous quantum-classical computing with nisq features," *ACM Trans. Quantum Comput.*, vol. 2, no. 4, pp. 1–37, 2021.
[24] Y. Shi *et al.*, "Resource-efficient quantum computing by breaking abstractions," *Proc. IEEE*, vol. 108, no. 8, pp. 1353–1370, 2020.
[25] Z. Xin-gang, L. Ji, M. Jin, and Z. Ying, "An improved quantum particle swarm optimization algorithm for environmental economic dispatch," *Expert Syst. Appl.*, vol. 152, p. 113370, 2020.
[26] F. S. Gharehchopogh, "Quantum-inspired metaheuristic algorithms: comprehensive survey and classification," *Artif. Intell. Rev.*, vol. 56, no. 6, pp. 5479–5543, 2023.
[27] A. Sadeghi Hesar and M. Houshmand, "A memetic quantum-inspired genetic algorithm based on tabu search," *Evol. Intell.*, pp. 1–17, 2023.
[28] S. Sengupta, S. Basak, and R. A. Peters, "Particle Swarm Optimization: A survey of historical and recent developments with hybridization perspectives," *Mach. Learn. Knowl. Extr.*, vol. 1, no. 1, pp. 157–191, 2018.
[29] H. M. H. Saad, R. K. Chakrabortty, S. Elsayed, and M. J. Ryan, "Quantum-inspired genetic algorithm for resource-constrained project-scheduling," *IEEE Access*, vol. 9, pp. 38488–38502, 2021.
[30] A. Ajagekar, K. Al Hamoud, and F. You, "Hybrid classical-quantum optimization techniques for solving mixed-integer programming problems in production scheduling," *IEEE Trans. Quantum Eng.*, vol. 3, pp. 1–16, 2022.
[31] K. K. Soni and A. Rasool, "Quantum Computation Based Grover's Search Algorithm and its Variations," *Recent Patents Eng.*, vol. 15, no. 4, pp. 45–54, 2021.
[32] E. Osaba *et al.*, "A tutorial on the design, experimentation and application of metaheuristic algorithms to real-world optimization problems," *Swarm Evol. Comput.*, vol. 64, p. 100888, 2021.
[33] J. Lang, S. Zielinski, and S. Feld, "Strategic portfolio optimization using simulated, digital, and quantum annealing," *Appl. Sci.*, vol. 12, no. 23, p. 12288, 2022.
[34] E. K. Grant, "Benchmarks and Controls for Optimization with Quantum Annealing," 2020.
[35] M. Cerezo *et al.*, "Variational quantum algorithms," *Nat. Rev. Phys.*, vol. 3, no. 9, pp. 625–644, 2021.
[36] A. P. Adelomou, E. G. Ribe, and X. V. Cardona, "Using the Parameterized Quantum Circuit combined with Variational-Quantum-Eigensolver (VQE) to create an Intelligent social workers' schedule problem solver," *arXiv Prepr. arXiv2010.05863*, 2020.
[37] A. Ajagekar, T. Humble, and F. You, "Quantum computing based hybrid solution strategies for large-scale discrete-continuous optimization problems," *Comput. Chem. Eng.*, vol. 132, p. 106630, 2020.
[38] N. J. Guerrero, "Solving Combinatorial Optimization Problems using the Quantum Approximation Optimization Algorithm," 2020.
[39] S. Khairy, R. Shaydulin, L. Cincio, Y. Alexeev, and P. Balaprakash, "Learning to optimize variational quantum circuits to solve combinatorial problems," in *Proceedings of the AAAI conference on artificial intelligence*, 2020, pp. 2367–2375.
[40] W. Zhang, H. He, and S. Zhang, "A novel multi-stage hybrid model with enhanced multi-population niche genetic algorithm: An application in credit scoring," *Expert Syst. Appl.*, vol. 121, pp. 221–232, 2019.
[41] S. Jerbi, L. M. Trenkwalder, H. P. Nautrup, H. J. Briegel, and V. Dunjko, "Quantum enhancements for deep reinforcement learning in large spaces," *PRX Quantum*, vol. 2, no. 1, p. 10328, 2021.
[42] M. H. Ullah, R. Eskandarpour, H. Zheng, and A. Khodaei, "Quantum computing for smart grid applications," *IET Gener. Transm. Distrib.*, vol. 16, no. 21, pp. 4239–4257, 2022.
[43] G. Verdon, J. Pye, and M. Broughton, "A universal training algorithm for quantum deep learning," *arXiv Prepr. arXiv1806.09729*, 2018.
[44] N. Moll *et al.*, "Quantum optimization using variational algorithms on near-term quantum devices," *Quantum Sci.*

ISSN 2986-2337 (Online)

Technol., vol. 3, no. 3, p. 30503, 2018.

[45] R. D. M. Simões, P. Huber, N. Meier, N. Smailov, R. M. Füchslin, and K. Stockinger, “Experimental evaluation of quantum machine learning algorithms,” *IEEE Access*, vol. 11, pp. 6197–6208, 2023.

[46] K. Egon, J. ROSINSKI, and L. KARL, “Quantum Machine Learning: The Confluence of Quantum Computing and AI,” 2023.

[47] A. Saxena, J. Mancilla, I. Montalban, and C. Pere, *Financial Modeling Using Quantum Computing: Design and manage quantum machine learning solutions for financial analysis and decision making*. Packt Publishing Ltd, 2023.